Full waveform microseismic inversion using differential evolution algorithm

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Outline

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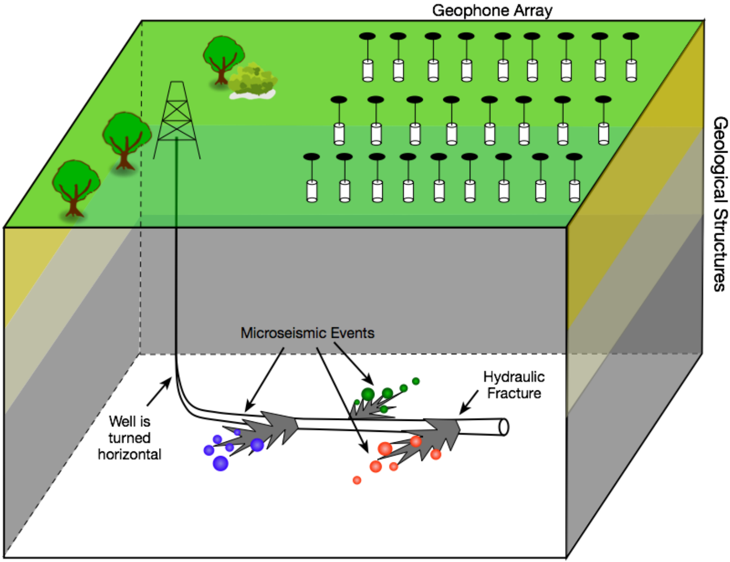
Surface monitoring during hydraulic fracturing



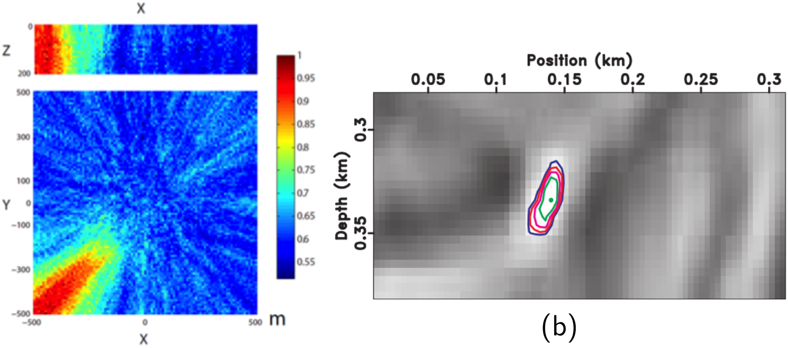
Summary

- Low oil price urges for cost-effective **long-term** monitoring
- Increasing interests on surface geophone array monitoring
  - *Low cost* comparing to wellbore array
  - Good azimuth angle coverage
  - *Long term* monitoring
- Microseismic events is a good indicator of subsurface structure changes
  - *Event location*
  - *Source mechanism*
- Processing pipeline
  - Pre-processing (QC and de-noise)
  - Event detection
  - **Event localization**
  - Finding **source mechanism**

Previous work on event localization

- Digitize the entire monitoring space into small blocks (grid nodes)
- Semblance (hours to days)
  [Gharti et al., 2010, Frantiek* et al., 2014]
  - Search all possible grid nodes using simple but fast method.
  - Rely on the coherent signal energy across the receiver array.
  - Low computation requirement, but might give misleading or imprecise results.
- Back-propagation (days to months)
  [Gajewski and Tessmer, 2005, Haldorsen et al., 2012]
  - Reverse time and back propagate wave field in digitized grids based on wave equations.
  - Take advantage of full waveform information.
  - Effective but expensive (time and memory), especially for 3D elastic wave.
  - Sensitive to model error, can have poor focusing.
- Both methods were developed using single component data

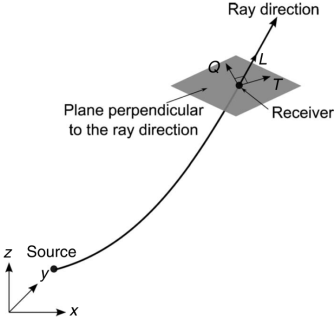
Example of traditional methods



(a) (b)

Figure 1: Event localization from (a)semblance based method and (b) reverse-time based method.

- 3-component(3-C) data is becoming popular
- Source mechanism is also important in reservoir monitoring
- Identify the source mechanism along with the localization becomes possible



(a) 3-C data

Moment tensor	Beachball	Moment tensor	Beachball
$\frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$		$-\frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$	
$-\frac{1}{\sqrt{2}}\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix}$		$\frac{1}{\sqrt{2}}\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}$	
$\frac{1}{\sqrt{2}}\begin{pmatrix}0&0&-1\\0&0&0\\-1&0&0\end{pmatrix}$		$\frac{1}{\sqrt{2}}\begin{pmatrix}0&0&0\\0&0&-1\\0&-1&0\end{pmatrix}$	
$\frac{1}{\sqrt{2}}\begin{pmatrix}-1&0&0\\0&0&0\\0&0&1\end{pmatrix}$		$\frac{1}{\sqrt{2}}\begin{pmatrix}0&0&0\\0&-1&0\\0&0&1\end{pmatrix}$	
$\frac{1}{\sqrt{6}}\begin{pmatrix}1&0&0\\0&-2&0\\0&0&1\end{pmatrix}$		$\frac{1}{\sqrt{6}}\begin{pmatrix}-2&0&0\\0&1&0\\0&0&1\end{pmatrix}$	
$\frac{1}{\sqrt{6}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}$		$-\frac{1}{\sqrt{6}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}$	

(b) Moment tensor

Outline

Problem formation

- Assumptions
  - Event origin time is given by event detection
  - Source waveform is available through wavelet estimation
  - Only **AWGN** is considered after pre-processing
  - **Isotropic** lossless layered velocity model
- Forward modelling of 3-C data
  - For complicated model, Finite-difference is used to compute Green's function
  - For layered velocity model, Green's function for *p-wave and s-wave* can be obtained by Generalized Ray Theory [Ben-Menahem and Singh, 2012]
  - Separate moment tensor and wave propagation due to **isotropy** of the media

Problem formation (cont.)

- Physical model
  - For i^{th} source and receiver pair, a Green's function $g[i]$ satisfies

    $$\boldsymbol{u}[i] = (g[i] * w)\boldsymbol{m}$$

    where $\boldsymbol{u}[i]$ is the data received, w is the source wavelet and $\boldsymbol{m}$ is the moment tensor.
  - Denote the convolution by $G[i] \triangleq g[i] * w$. Stack $G[i]$ into a big matrix $\boldsymbol{G}$ and data matrix $u[i]$ into $\boldsymbol{u}$, we have

    $$\boldsymbol{u} = \boldsymbol{G}\boldsymbol{m} \tag{1}$$

    where both $\boldsymbol{G}$ and $\boldsymbol{m}$ are unknown.
- For a set of receiver locations, fixed velocity model and source wavelet, G is only **a function of source location $\boldsymbol{s}$**, thus

$$\boldsymbol{u} = \boldsymbol{G}(\boldsymbol{s})\boldsymbol{m} \tag{2}$$

Minimization problem

- Original problem

$$\underset{\boldsymbol{s},\boldsymbol{m}}{\text{Minimize}} \ \|\boldsymbol{u} - \boldsymbol{G}(\boldsymbol{s})\boldsymbol{m}\| \tag{3}$$

- For a fixed $\boldsymbol{s}$, $\boldsymbol{m}$ can be estimated by least squares

$$\hat{\boldsymbol{m}}(\boldsymbol{s}) = (\boldsymbol{G}^H(\boldsymbol{s})\boldsymbol{G}(\boldsymbol{s}))^{-1}\boldsymbol{G}^H(\boldsymbol{s})\boldsymbol{u} \tag{4}$$

- New problem

$$\underset{\boldsymbol{s}}{\text{Minimize}} \ \boldsymbol{J}(\boldsymbol{s}) \triangleq \|\boldsymbol{u} - \boldsymbol{G}(\boldsymbol{s})\hat{\boldsymbol{m}}(\boldsymbol{s})\| \tag{5}$$

- In most cases, $\boldsymbol{J}(\boldsymbol{s})$ is a highly *non-linear, non-convex* function of $\boldsymbol{s}$.

Outline

Search for the minimum

- Grid search
  - Small model, coarse grid
  - Green's function of every source-receiver pair is evaluated
  - Minimum is guaranteed
- Differential Evolution algorithm (DE)
  - A smart way to sample the parameter space by population
  - Mutation is introduced for each generation(iteration) based on the current population
  - Selected mutants are compared with current population, the better one goes into the next generation
  - Requires fewer evaluations of forward modelling (computation of Green's function)

Differential evolution

- Initialization: randomly select an initial population of D agents consisting a set of parameters
- Mutation $\boldsymbol{v}_p$:

$$\boldsymbol{v}_p = \boldsymbol{x}_{p1} + F(\boldsymbol{x}_{p2} - \boldsymbol{x}_3) \tag{6}$$

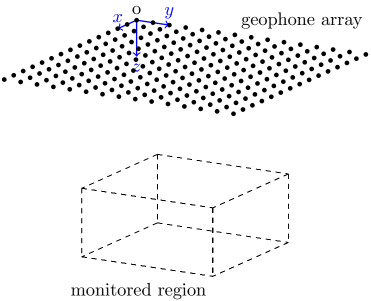
  where $F \in [0, 2]$, $\boldsymbol{x}_{p1}$ to $\boldsymbol{x}_{p3}$ are distinct and randomly selected from current population.
- Crossover:

$$u_j = \begin{cases} v_j & \text{if } p_j \leq C \text{ or } j = RI \\ x_j & \text{otherwise} \end{cases} \tag{7}$$

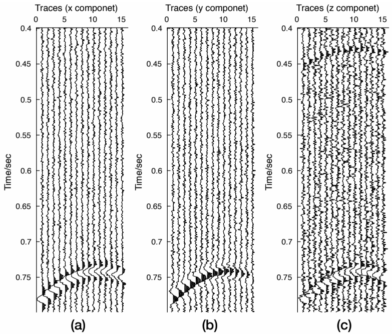
  where $p_j \sim U(0, 1)$, $C \in [0, 1]$, and random index(RI) is among $\{1, \cdots, D\}$.
- Selection: Choose between u_i and x_i and keep the one with lower cost function $\boldsymbol{J}(\boldsymbol{s})$.

Outline

Simulation setup



(i) Layout



(ii) Data with 25 dB PSNR

Figure 2: Simulation setup: (i)array geometry and (ii)sample data with 25dB PSNR: (a)x, (b)y, (c)z components.

Simulation setup

- 15 $\times$ 15 surface geophone array, double-couple moment sensor shown below:

$$MT = \begin{bmatrix} 0.4330 & -0.2500 & 0.7500 \\ -0.2500 & -0.4330 & 0.4330 \\ 0.7500 & 0.4330 & 0.0000 \end{bmatrix} \quad (8)$$

- Use PSNR as the measurement of noise level:

$$\text{PSNR} = 20 \log_{10} \frac{D_{\text{max}}}{\sigma} \quad (9)$$

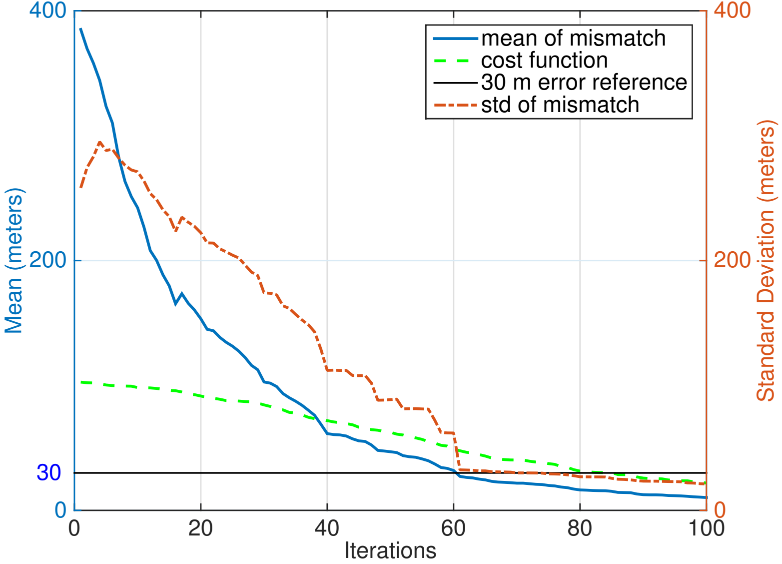
  where D_{max} is the maximum magnitude of a trace and σ is the standard deviation of AWGN.

- The model size is of 30 $\times$ 30 $\times$ 15 grid points with 40m spatial resolution

Details about DE algorithm

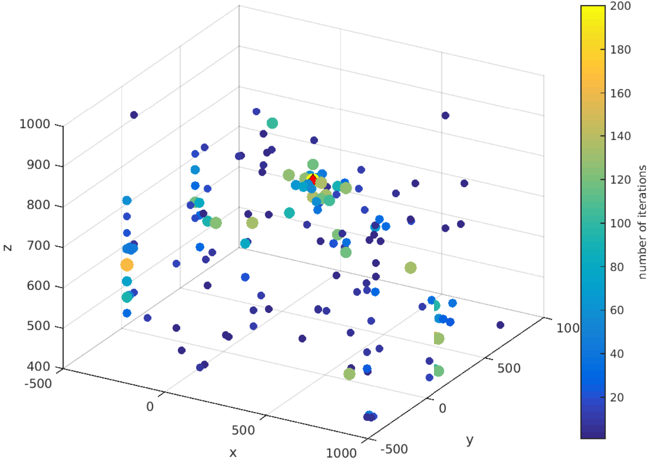
- Off-grid point
  - Move to the **nearest grid node**
  - Green's function of each node will only be evaluated once in the simulation
- Population size
  - Rule of thumb: population size is *5 to 10 times* the dimension of parameter space
  - In our example, the dimension of parameter space is three (x, y, z): **population size is 30**
- Accuracy measurement
  - The spatial resolution is 40m, the half diagonal distance is about 30m ($20\sqrt{2}$)
  - 60m error will be acceptable, **30m** error will be a good estimation
- Termination condition
  - DE program can be restarted at any iteration as long as the population is saved
  - Gradually increase the number of iteration until the cost function is stable

Convergence rate by iteration



Population convergence as iteration increases

- Population converges slower than the estimated error
- Dot color and size indicate number of iterations



Simulation results

- Accuracy
  - acceptable accuracy (60m error) within 40 iteration
  - good accuracy (30m error) within 60 iteration
- Robustness
  - Reach good accuracy in 100 iteration down to 0 dB PSNR
  - Event detection will break before the localization algorithm
- Computation requirement
  - Grid search: $30 \times 30 \times 15 \times 225 = 3,037,500$ evaluation of Green's function (4 days)
  - DE algorithm($C = 0.5$): $15 + 0.5 \times 30 \times 60 \times 225 = 205,875$ evaluation of Green's function (6 hours)
  - DE evaluates only 6.7% of all Green's functions

Conclusion

- The proposed method integrates **moment tensor inversion** and **event localization**
- **Reduce the dimension** of parameter space from 9 to 3 using proposed scheme
- Synthetic simulation illustrates a **good accuracy** of proposed method within reasonable time
- Differential evolution method **evaluates significantly fewer Green's functions** than grid search method

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Questions?

- Thanks for your attention!

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